Physics of Planetary Systems — Exercises — Set 2

Problem 2.1 *(2 points)*

Derive the following equation for the observed radial velocity amplitude of a companion on a circular orbit:

$$v_{\rm obs}[\mathrm{m/s}] = 28.4 \times \frac{m_{\rm p}[\mathscr{M}_{\rm Jup}] \sin i}{P[\mathrm{yr}]^{1/3} m_{\rm s}[\mathscr{M}_{\rm Sun}]^{2/3}}, \tag{1}$$

where $m_{\rm p}$ is the mass of the companion, i its orbital inclination with respect to the sky, P its orbital period, and $m_{\rm s}$ the mass of the host star.

Problem 2.2 *(2 points)*

You have discovered two new planets. Both have the same orbital period and inclination with respect to our line of sight, and both induce the same radial velocity amplitude (K_1 or $v_{\rm obs}$ amplitude). Their host stars have the same masses. One planet, however, is in a circular orbit and the other is in a highly eccentric orbit with $e = 0.9$. Which planet is the less massive and by how much?

Bonus problem 2.3 *(1 extra point)*

The gravitational potential energy of a homogeneous sphere of uniform mass density with radius R and mass $\mathscr{M}$ is

$$U = -\frac{3}{5}\frac{G\mathscr{M}^2}{R}. \tag{2}$$

Is the energy of a flat disk of the same radius and mass larger or smaller? Give arguments for your reasoning.

Problem 2.4 *(2 points)*

Derive the Jeans critical radius and mass more accurately by

- considering a homogeneous sphere of uniform density with radius R (instead of an arbitrarily shaped cloud of characteristic size R),
- using $v^2 = 3kT/(\mu m_p)$ (instead of $v^2 \sim kT/m_p$),
- employing the stability limit from the virial theorem, $K < |U|/2$ (instead of $K < |U|$).

Hint: use equation (2).

Problem 2.5 *(2 points)*

An initially large and cold spherical cloud of molecular hydrogen with $\mathscr{M} = \mathscr{M}_{\rm Sun}$ collapses to the radius $R = R_{\rm disk} = 200$ au, remaining spherical and homogeneous. Imagine the cloud is perfectly isolated from the outer world, so that no radiation comes out. Estimate the temperature T in its final state.

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